Solutions to JEE Advanced Home Practice Test -1 |JEE 2024 | Paper-1

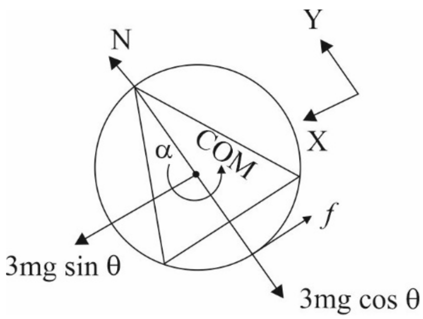
PHYSICS

MULTIPLE CHOICE

1.(B) $\sum F_x = ma$

$\Rightarrow \quad 3mg\sin\theta - f = 3ma$ ...... (i)

$\sum F_y = 0$



$\Rightarrow \quad N - 3mg\cos\theta = 0$ ...... (ii)

$\sum \tau = fR = l\alpha$ ...... (iii)

For no slipping $a = R\alpha$ ...... (iv)

The moment of inertia of the assembly about its centre of mass is $I = \frac{3}{2}mR^2$

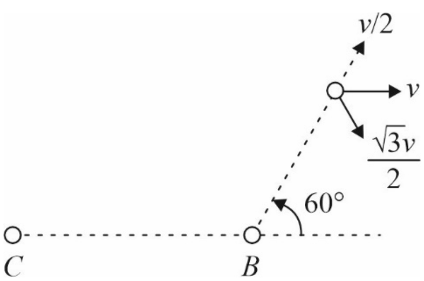
Now, on solving Eqs. (i), (ii), (iii) and (iv) simultaneously, we obtain $f = mg\sin\theta$

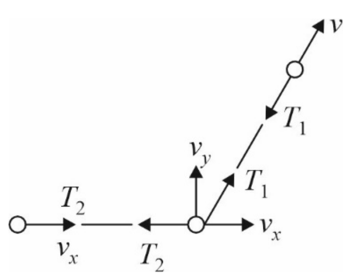
If μ is the coefficient of friction at the contact surface, then $f \le \mu N$

or $f \le \mu \times 3mg\cos\theta$ or $mg\sin\theta \le 3\mu\, mg\cos\theta$ or $\mu \ge \frac{1}{3}\tan\theta \quad \therefore \quad \mu_{min} = \frac{1}{3}\tan\theta$

2.(A) $\tau\omega t = JmS\Delta\theta(\omega = 2\pi n)$

3.(D) **Before** **After**





$-T_1\Delta t = (v_x\cos 60° + v_y\cos 30°) - \frac{v}{2}$ ...... (i)

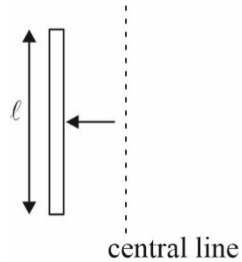
$T_1\Delta t\cos 30° = v_y$ ...... (ii)

$(T_1\cos 60° - T_2)\Delta t = v_x$ ...... (iii)

$T_2\Delta t = 2v_x$ ...... (iv)

Solving $v_x = \frac{v}{22}$

4.(A) $d\vec{F} = \frac{\mu_0 J \vec{r}}{2}(JdA)\ell$



$$\vec{F} = \int_0^a \frac{\mu_0 J \vec{r}}{2} JdA\ell = \frac{\mu_0 J^2 \ell}{2}\int_0^a \vec{r}\, dA = \frac{\mu_0 J^2 \ell}{2}\frac{\pi a^2}{2}\frac{4a}{3\pi} = \frac{\mu_0 J^2 a^3 \ell}{3}$$

5.(A) Path difference in air at point O, is given as

$$\Delta x = \left[\left(S_1O - t\right)n_2 + tn_3 - \left(S_2O\right)\right]t$$

$$\Rightarrow \quad \Delta x = \left[\left(S_1O - S_2O\right)n_2 + \left(n_3 - n_2\right)t\right] \quad \Rightarrow \quad \Delta x = \left(n_3 - n_2\right)t$$

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda_a} \times$ path difference in air,

$$\Delta\phi = \frac{2\pi}{n_1\lambda_1}\left(n_3 - n_2\right)t \quad \left(\because n_1 = \frac{\lambda_a}{\lambda}\right)$$

6.(A) $P + \rho_1 g h_1 = P + \rho_2 g h_2 \quad \therefore \quad \rho_2 = \frac{\rho_1 h_1}{h_2} = \frac{1000 \times 20}{10} = 2000 kg/m^3$

ONE OR MORE THAN ONE CHOICE

7.(BD) We use the given potential energy for calculation of force electron as

$$F = \frac{dU}{dr} = \frac{3}{2}\frac{Ke^2}{r^4} \quad \Rightarrow \quad KE = \frac{1}{2}mv^2 = \frac{3}{4}\frac{Ke^2}{r^3}$$

Using Bohr's II Postulate $mvr = \frac{nh}{2\pi}$ we get

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{3}{2}\frac{Ke^2}{2\pi} \quad \Rightarrow \quad r \propto \frac{1}{n^2} \text{ and } r \propto m$$

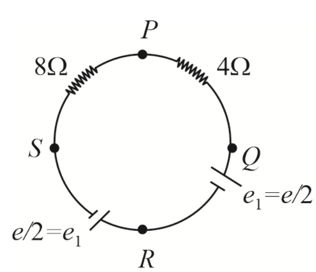
Hence option (B) and (D) are correct as energy is inversely proportional to r^3.

8.(AC) Electric equivalent:

$$i = \frac{e_1 + e_2}{4 + 8} \Rightarrow \frac{e}{12} = 4$$

$$e = 48V$$

$$\Delta V_{PR} = e_1 - i(4) = 24 - 4(4) = 8V$$



9.(ABCD) $\left[L'T'^{-1}\right]=\frac{\alpha^2}{\beta}\left[LT^{-1}\right]$ ...... (1)

$\left[L'T'^{-2}\right]=\alpha\beta\left[LT^{-2}\right]$ ...... (2)

$\left[M'L'T'^{-2}\right]=\frac{1}{\alpha\beta}\left[MLT^{-2}\right]$ ...... (3)

Dividing eq. (3) by (2), we get, $\left[M'\right]=\frac{1}{\alpha^2\beta^2}\left[M\right]$

Dividing eq. (1) by (2), we get $\left[T'\right]=\frac{\alpha^2}{\beta}\times\frac{1}{\alpha\beta}\left[T\right]=\frac{\alpha}{\beta^2}\left[T\right]$

Now, $\frac{\left[eq.(1)\right]^2}{eq.(2)}$ gives

$$\frac{\left[L'T'^{-1}\right]}{\left[L'T'^{-2}\right]}=\frac{\alpha^4}{\beta^2}\times\frac{1}{\alpha\beta}\frac{\left[LT^{-1}\right]^2}{\left[LT^{-2}\right]} \quad\Rightarrow\quad \left[L'\right]=\frac{\alpha^3}{\beta^3}\left[L\right]$$

$$\frac{p'}{p}=\frac{\left[M'L'\right]}{\left[T'\right]}\frac{\left[T\right]}{\left[ML\right]}=\frac{1}{\alpha^2\beta^2}\times\frac{\alpha^3}{\beta^3}\times\frac{\beta^2}{\alpha}=\frac{1}{\beta^3}$$

10.(ABD) $\frac{du}{dx}=0 \quad\Rightarrow\quad x=\pm\sqrt{\frac{b^2-4a^2}{3}}$

For $b=2.5a$ two positions exist where $E=0$.

$$u_{\min}=a\left[\frac{2kq}{\sqrt{a^2+\frac{b^2+4a^2}{3}}}-\frac{16kq}{\sqrt{b^2+\left(\frac{b^2-4a^2}{3}\right)}}\right]=\frac{-6\sqrt{3}kq^2}{\sqrt{b^2-a^2}}$$

$u_\infty=0$; Potential at $V_0=2kq\left[\frac{1}{a}-\frac{8}{b}\right]$

For $b=4a$, $V_0<0$ so min velocity of charge q is zero

For $b=16a$ using conservation of energy $\frac{1}{2}mv^2+0=0+qV_0 \quad\Rightarrow\quad v_{\min}=\sqrt{\frac{kq^2}{ma}}$

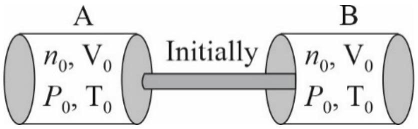
11.(AD) $R=\frac{\rho l}{A}$, $\rho=\frac{RA}{l}=1.6\times10^{-8}\,\Omega\text{-}m$

Current density using $j=\frac{I}{A}=2.4\times10^5\,A/m^2$

Number density of charge carriers as $n=\frac{j}{v_d e}=8.8\times10^{28}\,m^{-3}$

12.(BC) Initially for container A, $P_0V_0 = n_0RT_0$

For container B, $P_0V_0 = n_0RT_0$ $\therefore$ $n_0 = \dfrac{P_0V_0}{RT_0}$



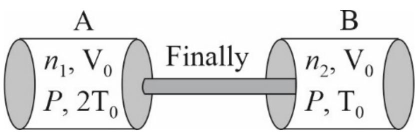
Total number of moles $= n_0 + n_0 = 2n_0$

Since even on heating, the total number of moles is conserved,

We have $n_1 + n_2 = 2n_0$ ...... (i)

Let P be the common pressure, Then for container A,

$PV_0 = n_1R\,2T_0$ $\therefore$ $n_1 = \dfrac{PV_0}{2RT_0}$



And for container B, $PV_0 = n_2RT_0$ $\therefore$ $n_2 = \dfrac{PV_0}{RT_0}$

Substituting the values of n_0, n_1 and n_2 in equation (i), we get

$$\frac{PV_0}{2RT_0} + \frac{PV_0}{RT_0} = \frac{2.P_0V_0}{RT_0} \Rightarrow P = \frac{4}{3}P_0$$

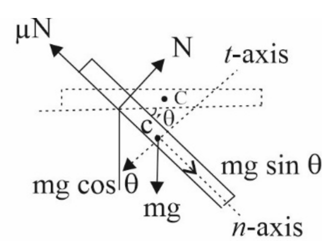
Number of moles in container A (at temperature $2T_0$)

$$= n_1 = \frac{PV_0}{2RT_0} = \left(\frac{4}{3}P_0\right)\frac{V_0}{2RT_0} = \frac{2}{3}\frac{P_0V_0}{RT_0} \quad \left[As\ P = \frac{4}{3}P_0\right]$$

NUMERICAL VALUE TYPE

13.(2) From the law of conservation of energy,

$KE_i + GPE_i = KE_f + GPE_f$



$$\therefore \quad 0 + 0 = \frac{1}{2}I\omega^2 + \left(-mg\frac{a}{3}\sin\theta\right) \quad ...(i)$$

From parallel axes theorem, $I = \dfrac{m(2a)^2}{12} + m\left(\dfrac{a}{3}\right)^2$

Using this in eq. (i) will get $\omega^2 = \dfrac{3g\sin\theta}{2a}$

Differentiating w. r. t. θ, we get

$$2\omega\frac{d\omega}{d\theta}=\frac{3\omega}{2a}\cos\theta$$

$$\alpha=\omega\frac{d\omega}{d\theta},\ \alpha=\frac{3g}{4a}\cos\theta$$

Along t-axis, $\sum F_t = mg\cos\theta - N = m\frac{a}{3}\alpha$

or $\quad mg\cos\theta - N = m.\frac{a}{3}.\frac{3g}{4a}\cos\theta \quad\Rightarrow\quad N=\frac{3mg}{4}\cos\theta$

Along n-axis, $\sum F_n = \mu N - mg\sin\theta = m\left(\frac{a}{3}\right)\left(\omega^2\right)$

$$\mu\left[\frac{3mg}{4},\cos\theta\right]-mg\sin\theta = m\left(\frac{a}{3}\right)\left[\frac{3g\sin\theta}{2a}\right]$$

$$\mu\frac{3mg}{4}\cos\theta=\frac{3}{2}mg\sin\theta \quad\Rightarrow\quad \mu=2\tan\theta$$

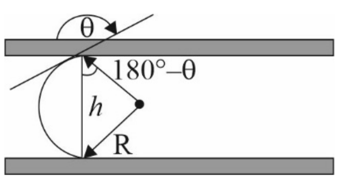
14.(2) Pressure inside a film greater than outside pressure by an amount $T\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$. If θ is the angle of contact then $h=2r_1\cos(\pi-\theta)$ or $r_1=-\frac{h}{2\cos\theta}$. Since the tablet is between the plates, so $r_2=R$.

Thus pressure difference $=T\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=T\left[-\frac{1}{h/2\cos\theta}+\frac{1}{R}\right]$

As h is small in comparison to R, so $\frac{1}{R}<<\frac{1}{h} \quad \therefore \quad P=-\frac{2T\cos\theta}{h}$

The total force exerted by mercury drop on the upper glass plate is nearly

$$F = \text{p}\times\text{projected area of drop} = \left(-\frac{2T\cos\theta}{h}\right)\times\pi R^2 = -\frac{2\pi R^2 T\cos\theta}{h} \qquad \text{...... (i)}$$



Let R′ becomes the new radius of curvature when the distance between the plates is decreased by n-times. Assuming mercury to be incompressible, we have

$$\pi R^2 h = \pi R'^2\left(\frac{h}{n}\right) \quad\Rightarrow\quad R'=\sqrt{n}R$$

The force exerted by the mercury drop now becomes

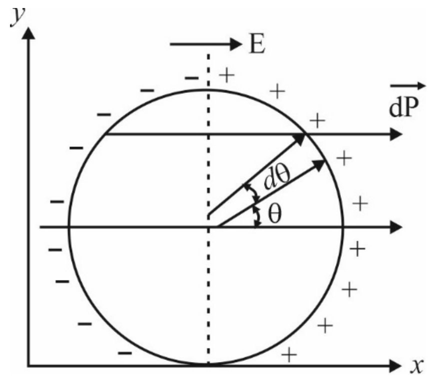
$$F' = -\frac{2\pi\left(\sqrt{n}R\right)^2 T\cos\theta}{\left(h/n\right)} = n^2F \qquad \text{...... (ii)}$$

If mg be the weight placed on the upper plate then $F'=F+mg$

$$\therefore \quad m=\frac{F'-F}{g}=\frac{F\left(n^2-1\right)}{g}=\frac{2\pi R^2 T\cos\theta}{gh}\left(1-n^2\right)$$

15.(1) $\lambda = \frac{q}{\pi r}$

The dipole moment of the ring $P = \int_{-\pi/2}^{\pi/2} (\lambda r d\theta . 2r\cos\theta)$



$P = 4r^2 l = \frac{4qr}{\pi}$

$\boxed{P = \frac{4qr}{\pi}}$

$\vec{\tau} = \vec{P} \times \vec{E}$ For small angle $\theta \sin\theta \simeq \theta$

$\tau = PE\sin\theta$

$\frac{mr^2}{2}\frac{d^2\theta}{dt^2} = \frac{4qr}{\pi}\theta$

$\frac{d^2\theta}{dt^2} = \frac{8qE}{\pi mr}\theta$

$\boxed{\frac{d^2\theta}{dt^2} \alpha\ \theta}$ $\Rightarrow$ $\omega = \sqrt{\frac{8qE}{\pi mr}}$ $\Rightarrow$ $T = \frac{2\pi}{\omega} = \pi\sqrt{\frac{\pi mr}{2qE}}$

16.(5.50) If T_B be the temperature at B, then by gas law $\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B}$

$\therefore$ $T_B = \frac{P_B V_B}{P_A V_A} T_A = \frac{(2P_0)(2V_0)}{P_0 V_0} = T_0$

The change in internal energy from A to B

$\Delta U = nC_v \Delta T = 1 \times \frac{3R}{2} \times (4T_0 - T_0) = \frac{9RT_0}{2}$

Work done in the process A to C $W_{AC} = P\Delta V = P_0(2V_0 - V_0) = P_0 V_0 = RT_0$ and $W_{CB} = 0$

$\therefore$ Total work done from $A \to C \to B$

$W_{AC} + W_{CB} = RT_0 + 0 = RT_0$

From the first law of thermodynamics, Q = $\Delta U + W = \frac{9RT_0}{2} + RT_0 = \frac{11RT_0}{2}$

Thus, heat absorbed by the gas from $A \to C \to B$ is $\frac{11RT_0}{2}$

17.(0.5) The wavelength of the sound from the tuning fork is $\lambda = \frac{v}{f}$. The cylinder is a pipe open at the top and closed at the water surface; its resonance patterns are AN, ANAN, ANANAN, etc. Resonance occurs each time the height of the air column changes by half a wavelength: $\Delta h = \frac{v}{2f}$. The volume of the pipe between these two water levels is $\pi r^2 \Delta h,$ which is also equal to the amount of water that has entered the pipe at rate R in a time interval Δt and has filled this volume.

Therefore, $R\Delta t = \pi r^2 \Delta h = \frac{\pi r^2 h}{2f} \Rightarrow \Delta t = \frac{\pi r^2 v}{2Rf}$

18.(2.0) Total charge on disc $q_0 = \int_0^R \sigma_0 \left(1 - \frac{r}{R}\right) 2\pi r dr = \sigma.2\pi\left[\int_0^R r dr - \frac{1}{R}\int_0^R r^2 dr\right] = 2\pi\sigma\left[\frac{R^2}{2} - \frac{R^2}{3}\right]$

$$q_0 = 2\pi\sigma\left[\frac{R^2}{6}\right] \quad \text{...(i)}$$

Charge enclosed by second sphere

$$q = \sigma.2\pi\left[\int_0^{R/2} r dr - \frac{1}{R}\int_0^{R/2} r^2 dr\right]$$

$$q = \frac{2\pi\sigma R^2}{12}; \quad \frac{\phi_0}{\phi} = \frac{q_0}{q} = \frac{12}{6} = 2$$

CHEMISTRY

MULTIPLE CHOICE

1.(B) For unsymmetrical distribution curve $V_{mps} < V_{avg} < V_{rms}$

2.(D) $Na_2CO_3 \xrightarrow{\Delta} X$ (No decomposition due to thermal stability).

3.(D) $X_2 = O_2$ $X_3 = O_3$ $Y_2 = I_2$ $\left(I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}\right)$

4.(B) **(A)** Identical, **(B)** Diastereomers
(C) Positional isomer **(D)** Identical

5.(B) Fact

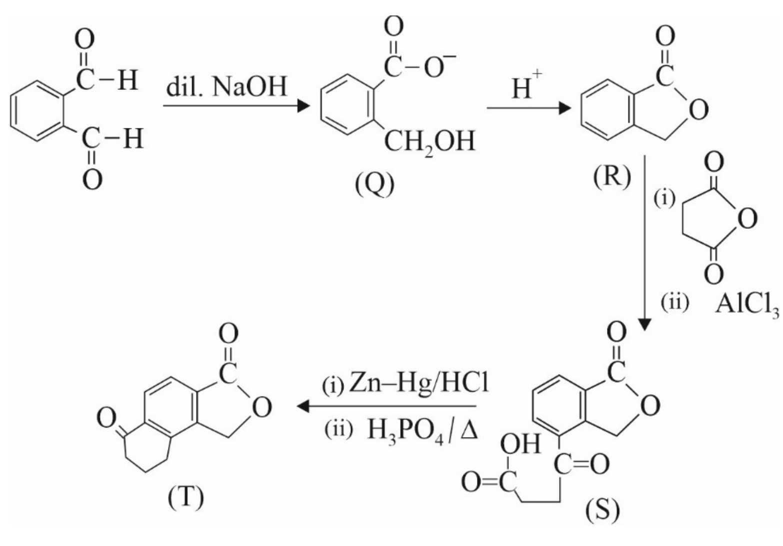
6.(B) No free reducing site is present due to glycosidic linkage $C_1 - C_2$

ONE OR MORE THAN ONE CHOICE

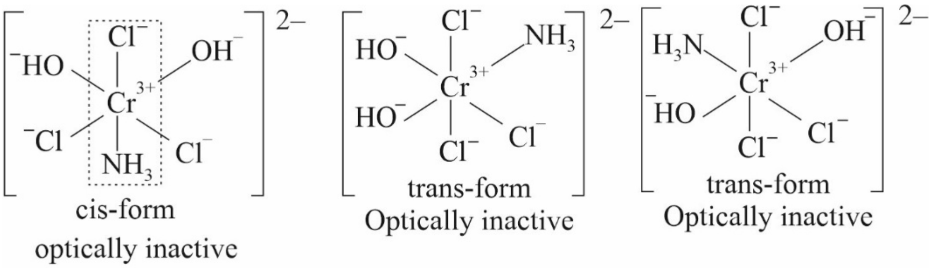
7.(ABC)
Here, D option is the work done for reversible isothermal process.

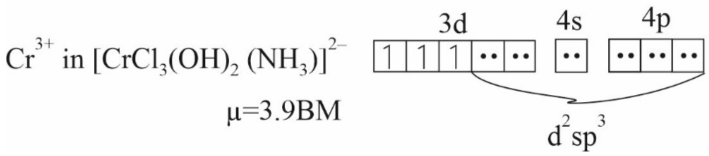
8.(ABCD)
In imidazole l.p. of N—1 is delocalised;
Purine l.p. of N—9 is delocalised;
Pyrimidine $6\pi e^-$ system;
Imidazole $6\pi e^-$ system purine $10\pi e^-$ system.

9.(AC)



10.(AD)





11.(BD)

(B) $HClO_4 + H_2O \rightarrow H_3O^+ + ClO_4^-$ since H_2O is accepting H^+ from $HClO_4$ so H_2O is stronger base compared to ClO_4^-.

(C) perchlorate do not participate in disproportionation reaction.

(D) $ClO^- + NO_2^- \rightarrow Cl^- + NO_3^-$

12.(ABC)

NUMERICAL VALUE TYPE

13.(1.0)

(I) Phenolphthalein indicates partial neutralisation of $Na_2CO_3 \longrightarrow NaHCO_3$

meq. Of $Na_2CO_3 +$ meq. of NaOH = meq. of HCl

$$\frac{W}{E}\times 1000 + \frac{W}{E}\times 1000 = NV$$

(Suppose $Na_2CO_3 = a$ gm, $NaOH = b$ gm)

$$\frac{a}{106}\times 1000 + \frac{b}{40}\times 1000 = 300\times 0.1 \qquad ...(1)$$

(II) Methyl orange indicates complete neutralisation

$N_1V_1 = N_2V_2$

$25\times 0.2 = 0.1\times V_2$ so $V_2 = 50$ mL excess

$$\therefore \quad \frac{a}{53}\times 1000 + \frac{b}{40}\times 1000 = 350\times 0.1 \qquad ...(2)$$

From eqns. (1) and (2). b = 1g

14.(532) $K_p^0 = \frac{P_{CO_2}}{P_{CO}} = \frac{10^6}{400}$

$$\text{Now}, \Delta G^\circ = -5320 - 5.6T = -RT\ln K_p^0 = -2\times T\times \ln\frac{10^6}{400} \quad \therefore \quad T = 532K$$

15.(68.53)

I. $\frac{1}{2}O_2(g)+2H^+ +2e^- \longrightarrow H_2O(\ell), \quad E^0 = 1.23V$

II. $H_2(g) \longrightarrow 2H^+ + 2e^-, E° = 0.00V$

$H_2(g)+\frac{1}{2}O_2(g) \longrightarrow H_2O, \quad E° = 1.23V$

$\Delta G° = -2\times 96500\times 1.23\times 0.1\times 0.8 = -189.912J$

$W = 189.912J$

$W_{adiabatic} = \frac{nR}{(\gamma-1)}(T_2 - T_1)$

$189.912 = \frac{1\times 8.314}{\left(\frac{4}{3}-1\right)}(T_2 - T_1)$; $\Delta T = \frac{189.912}{8.314}\times\frac{3}{1} = 68.53$

16.(4.89) $Mg \quad + \quad H_2SO_4 \rightarrow MgSO_4 + H_2$

$\frac{4.8}{24} = 0.2 \quad 0.25 \qquad\qquad 0.2mol$

$V_{H_2} = \frac{0.2\times 0.0821\times 298}{1} = 4.89L$

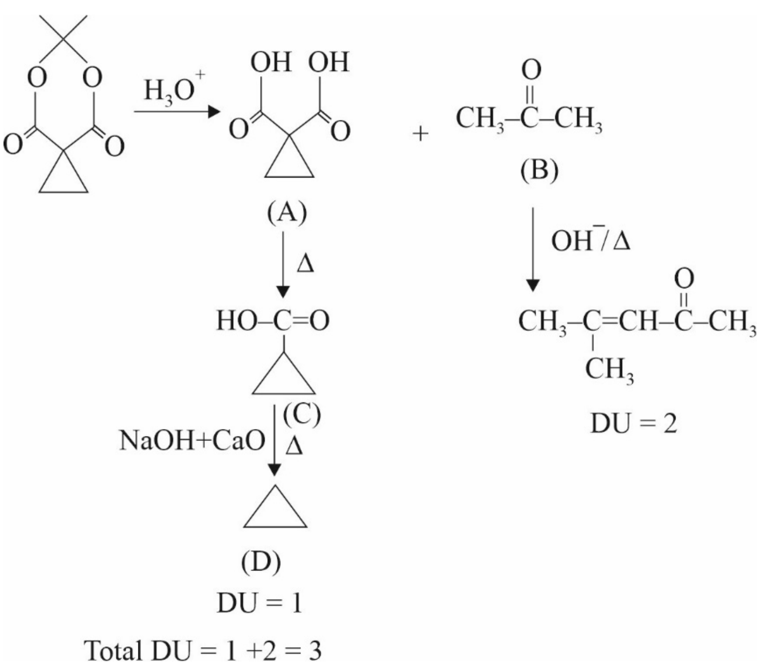
17.(247.5) ${}_{84}Po^{218} \longrightarrow {}_{82}Pb^{214} + {}_2He^4 \longrightarrow {}_{83}Bi^{214} + {}_{-1}e^0$

Pb^{214} to reach max. no. of nuclei

$t_{max.} = \frac{1}{\lambda_1-\lambda_2}\ln\frac{\lambda_1}{\lambda_2} = 247.5\,sec$

Where $\lambda_1 = \frac{0.693}{183}; \lambda_2 = \frac{0.693}{161}$

18.(3)



MATHEMATICS

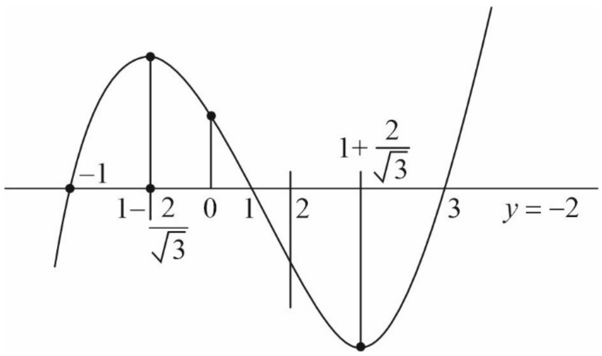
MULTIPLE CHOICE

1.(C) $a^2(c+d)+c^2[a+b]+b^2(c+d)+d^2(b+a)$

$= 22\left[a^2+b^2-c^2-d^2\right] = 22\left[(a+b)^2-2ab-(c+d)^2+2cd\right]$

$= 22[484+4040-484+4040] = 8080\times 22 = 177760$

2. (A)



$f(x)=x^3-3x^2-x+1$; $\quad f(x)=(x+1)(x-1)(x-3)-2$

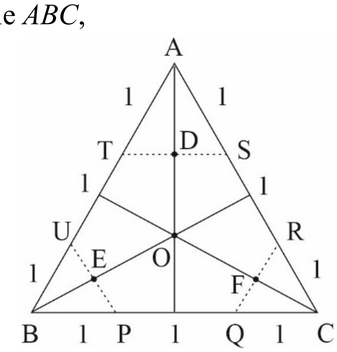
$f'(x)=0$; $\quad 3x^2-6x-1=0$

$x=1\pm\dfrac{2}{\sqrt{3}}$; $\quad f(0)=1$

$f(4)=13$; $\quad f(2)=-5$

$f(-2)=-17 \Rightarrow$ $[a, b]$ may be [–1, 1] or [–1, 3] or [1, 3]; Sum = 0 + 2 + 4 = 6

3.(C) The first task is to identify the figure that results after folding. In the figure below, we show the original triangle *ABC*,



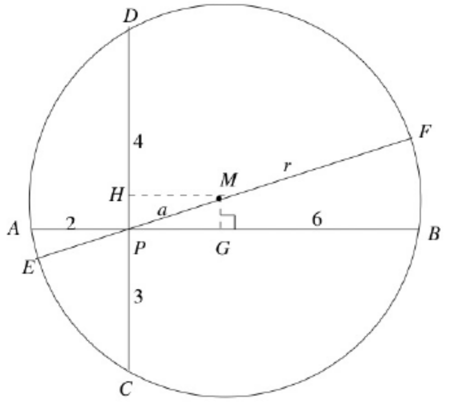
As ABC is an equilateral triangle with side length 3, each of its medians has length $\dfrac{3\sqrt{3}}{2}$

Hence OC equals $\dfrac{2}{3}\times\dfrac{\sqrt{3}}{2}\times 3=\sqrt{3}$. Hence $FC=\dfrac{\sqrt{3}}{2}$

Therefore $QR = 2FR = 2FC\tan 30^\circ = \sqrt{3}\times\dfrac{1}{\sqrt{3}}=1$

Area required $=\dfrac{9\sqrt{3}}{4}-\dfrac{3\sqrt{3}}{4}=\dfrac{3\sqrt{3}}{2}$

4.(B) We know that $PA \cdot PB = PC \cdot PD \quad \Rightarrow \quad PD = 4$



To find *r*, the radius of the circle we let *M* be its centre and *a* its distance from *P*. Then applying the theorem to the diametrical chord *EF* through P, we get

$12 = EP \cdot PF = (EM - PM) \cdot (PM + MF) = (r-a)(r+a) = r^2 - a^2$ ...... (1)

So we would know *r* if we can find a, i.e. *PM*. For this, drop perpendiculars *MG*, *HM* from *M* to the chords *AB* and *CD* respectively. Then *G* and *H* are the midpoints of *AB* and *CD* respectively.

So $PG = AG - AP = 4 - 2 = 2$ and similarly, $PH = PD - HD = 4 - \frac{7}{2} = \frac{1}{2}$

Hence from the right-angled triangle *PGM*,

$a^2 = PM^2 = PG^2 + GM^2 = PG^2 + PH^2 = 4 + \frac{1}{4} = \frac{17}{4}$

Putting this into (1) gives $r^2 = 12 + \frac{17}{4} = \frac{65}{4}$, Hence $r = \frac{\sqrt{65}}{2}$

5.(C) We have $a^2 = 9, b^2 = 5$

$\therefore \quad e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{5}{9} = \frac{4}{9} \quad \Rightarrow \quad e = \frac{2}{3}$

Thus, $f_1 = ae = 2$ and $f_2 = -2$

An equation of P_1 is $y^2 = f_1 x = 8x$ and an equation of P_2 is $y^2 = -16x$

Let equation of T_1 be $y = m_1 x + \frac{2}{m_1}$

As T_1 passes through $(2f_2, 0) = (-4, 0)$; $\quad 0 = -4m_1 + \frac{2}{m_1} \quad \Rightarrow \quad m_1^2 = \frac{1}{2}$

An equation of T_2 is $y = m_2 x - \frac{4}{m_2}$

As it passes through $(f_1, 0) = (2, 0)$

$0 = 2m_2 - \frac{4}{m_2} \quad \Rightarrow \quad m_2^2 = 2$

Thus, $\frac{1}{m_1^2} + m_2^2 = 2 + 2 = 4$

6.(A) $P(E)=\frac{13}{96}$

$$\frac{1}{9}+2\left(\frac{1}{36}-x^2\right)=\frac{13}{96}$$

Simplifying, $\frac{1}{36}-x^2=\frac{1}{2}\left(\frac{13}{96}-\frac{1}{9}\right)=\frac{1}{2}\times\frac{39-32}{288}=\frac{7}{576}$

Hence $x^2=\frac{1}{36}-\frac{7}{576}=\frac{16-7}{576}=\frac{9}{576}=\frac{1}{64}$

Which gives $x=\frac{1}{8}$

ONE OR MORE THAN ONE CHOICE

7.(ABD)

Consider the function $h(x)=f(x)-kg(x)$ where $k\in\{1, 2, 3, 4\}$ on the interval [0, 1]

Using LMVT, we get

$$h'(c)=\frac{h(1)-h(0)}{1-0}=f'(c)-kg'(c)$$

$$\left(f(1)-kg(1)\right)-\left(f(0)-kg(0)\right)=f'(c)-kg'(c)$$

$$(6-2k)-(2-0)=f'(c)-kg'(c)$$

$$4-2k=f'(c)-kg'(c)$$

Now, if $k=1,$ we get $f'(c)-g'(c)=2=f(0)$ $\Rightarrow$ (A) is true

If $k=2$

$f'(c)-2g'(c)=0=g(0)$ $\Rightarrow$ (B) is true

If $k=3$

$f'(c)-3g'(c)=-2=-g(1)$ $\Rightarrow$ (C) is false

If $k=4$

$f'(c)-4g'(c)=-4=-2g(1)$ $\Rightarrow$ (D) is true

8.(ACD)

$$adj\left(adj\left(adj(M)\right)\right)=|M|^{(n-2)(n-1)}\,adj(M)$$

$$M^{-1}=\frac{1}{|M|}adj\ M$$

$\Rightarrow$ $|M|M^{-1}=adj\ M$

$|M|M^{-1}adj\left(M^{-1}\right)=(adjM)\,adj\left(M^{-1}\right)=adj\left(M^{-1}M\right)=I$

$\Rightarrow$ $adj(kA)=k^{n-1}adj(A)$

9.(ABC) Both the sets consist of reciprocals of points on a circle. But in A, since $\left|\frac{1}{z}\right| = \frac{1}{|z|}$, A is nothing but $\left\{z : |z| = \frac{1}{2}\right\}$ which is clearly a circle. In B, however, although all points of the starting circle are equidistant from 1, their reciprocals need not

For B let $\frac{1}{z} = x + yi$ and given $|z-1| = 2$

$$\Rightarrow \quad \left|\frac{1}{x+yi} - 1\right| = 2 \quad \Rightarrow \quad \left(\frac{x}{x^2+y^2} - 1\right)^2 + \left(\frac{y}{x^2+y^2}\right)^2 = 4$$

$$\Rightarrow \quad \frac{x^2+y^2}{\left(x^2+y^2\right)^2} - \frac{2x}{x^2+y^2} = 3 \quad \Rightarrow \quad 1 - 2x = 3\left(x^2+y^2\right)$$

So B is a circle with radius $= \frac{2}{3}$, centre $\left(-\frac{1}{3}, 0\right)$

10.(ABD) We have $a^2 + b^2 + c^2 = ca + ab\sqrt{3}$,

$$\Rightarrow \quad a^2 + b^2 + c^2 - ca - ab\sqrt{3} = 0 \quad \Rightarrow \quad \left(\frac{a\sqrt{3}}{2} - b\right)^2 + \left(\frac{a}{2} - c\right)^2 = 0$$

It is possible only when $\frac{a\sqrt{3}}{2} - b = 0$ and $\frac{a}{2} - c = 0$

$$\Rightarrow \quad \sqrt{3}a = 2b = 2c\sqrt{3} = k\,(\text{let}) \quad \Rightarrow \quad a = \frac{k}{\sqrt{3}},\ b = \frac{k}{2} \text{ and } c = \frac{k}{2\sqrt{3}}$$

$$\therefore \quad b^2 + c^2 = a^2 \angle A = 90°;\ \sin B = \frac{b}{a} = \frac{\sqrt{3}}{2} \qquad \therefore \quad \angle B = 60° \text{ and } \angle C = 30°.$$

11.(ABCD)

Three planes meet at two points it means they have infinitely many solutions, so $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ \alpha & -1 & 3 \end{vmatrix} = 0$

$$\Rightarrow \quad 2(-3+1) - 1(3+1) + \alpha(1+1) = 0 \quad \Rightarrow \quad \alpha = 4$$

$P_1 : 2x + y + z = 1$

$P_2 : x - y + z = 2$

$P_3 : 4x - y + 3z = 5$

P on XOY plane $\equiv (1, -1, 0)$

(Which can be obtained by putting $z = 0$ in any two of the given planes)

Q on YOZ plane $\equiv \left(0, \frac{-1}{2}, \frac{3}{2}\right)$

(Which can be obtained by putting $x = 0$ in any two of the given planes.)

$\therefore$ Straight line perpendicular to plane P_3 passing through P is $\dfrac{x-1}{4}=\dfrac{y+1}{-1}=\dfrac{z}{3}$

$\overrightarrow{PQ}=\hat{i}-\dfrac{1}{2}\hat{j}-\dfrac{3}{2}\hat{k}$

Projection of $\overrightarrow{PQ}$ on x-axis $\Rightarrow$ $\left|\dfrac{\overrightarrow{OP}.\hat{i}}{|\hat{i}|}\right|=1$

Centroid of ΔOPQ is $\left(\dfrac{1}{3},\dfrac{-1}{2},\dfrac{1}{2}\right)$

12.(AB) We know that $\cos^{-1}x=\dfrac{\pi}{2}-\sin^{-1}x,\ -1\le x\le 1,$

So, that $I(a)=I_1+I_2$

Where $I_1=\dfrac{\pi}{2}\int_{-1/a}^{1/a}\left(2x^6+2x^4+3\right)dx=\pi\int_0^{1/a}\left(2x^6+2x^4+3\right)dx$

$=\pi\left(\dfrac{2}{7}x^7+\dfrac{2}{5}x^5+3x\right)_0^{1/a}=\pi\left(\dfrac{2}{7a^7}+\dfrac{2}{5a^5}+\dfrac{3}{a}\right)$

and $I_2=\int_{-1/a}^{1/a}\left(2x^6+2x^4+3\right)\sin^{-1}(ax)\,dx=0$ [integrand is odd function]

Also $I(a)\le\dfrac{\pi}{a}\left(\dfrac{2}{7}+\dfrac{2}{5}+3\right)=\dfrac{129\pi}{35a}$

NUMERICAL VALUE TYPE

13.(140)

Let the size of the rectangular block be $x\times y\times z$.

Then we have $n=xyz$ …… (1)

$(x-2)(y-2)(z-2)=30$ …… (2)

The problem asks to minimise (1) subject to the constraint (2).

In only a few ways. We can try all such factorisatons, one by one, and calculate the value of n for each. For example, the factrorisation $6\times5\times1$ gives $x-2=6,\ y-2=5$ and $z-2=1$. Hence $x=8,\ y=7,\ z=3$. So n = 168. But intuitively, it is clear that to minimise n, the rectangular block should be as close to a cube as possible. 30 is not a perfect cube. But the factrorisation 5 × 3 × 2 is the closest we can come. For this factroisation, $n=7\times5\times4=140$.

14.(250)

$(1-x)^{200}={}^{200}C_0-{}^{200}C_1x+{}^{200}C_2x^2-{}^{200}C_3x^3+....$ …… (i)

and $\left(\dfrac{1-x^{149}}{1-x}\right)=\left(x^{148}+x^{147}+...+x+1\right)$ …… (ii)

On multiplying and equations coefficient of x^{148}

We get ${}^{200}C_0 - {}^{200}C_1 + {}^{200}C_2 .... + {}^{200}C_{148}$

$$= \text{Coefficient of } x^{148} \text{ in} \left[(1-x)^{200} . \frac{\left(1-x^{149}\right)}{(1-x)} \right]$$

$$= \text{Coefficient of } x^{148} \text{in} \left[(1-x)^{199} \left(1-x^{149}\right) \right]$$

$$= \text{Coefficient of } x^{148} \text{ in } (1-x)^{199} = {}^{199}C_{148} \text{ or } {}^{199}C_{51}$$

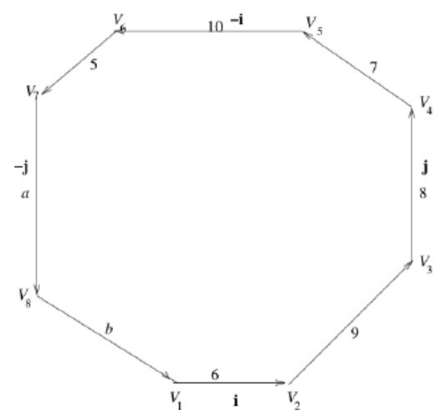
15.(4) $\cos 2x + 3\sin 2x = -3$

$$\frac{1}{\sqrt{10}}\cos 2x + \frac{3}{\sqrt{10}}\sin 2x = -\frac{3}{\sqrt{10}}$$

$\Rightarrow \quad \cos(2x-\alpha) = -\frac{3}{\sqrt{10}}$ where $\cos\alpha = \frac{1}{\sqrt{10}}$ and $0 \le 2x < 4\pi$

$\Rightarrow$ Four solutions of x

16.(18) Call the vertices of the octagon as $V_1, V_2, ..., V_8$. As all angles are equal, each of them is $\frac{12\times 90}{8} = 135°$. So, if we let i be a unit vector along V_1V_2, then the unit vectors along the subsequent sides are $\frac{i+j}{\sqrt{2}}, j, \frac{-i+j}{\sqrt{2}}, -i, -\frac{i+j}{\sqrt{2}}, -j$ and $\frac{i-j}{\sqrt{2}}$ in that order as shown in the diagram below.



Let a and b be the lengths of the sides V_7V_8 and V_8V_1 respectively. Then, the vector sum $\sum V_i\vec{V}_{i+1} + \overrightarrow{V_8V}_1$ is the zero vector.

This gives $6i + 9\frac{i+j}{\sqrt{2}} + 8j + 7\frac{-i+j}{\sqrt{2}} - 10i - 5\frac{i+j}{\sqrt{2}} - aj + b\frac{i-j}{\sqrt{2}} = 0$ ...... (1)

Equating the coefficients of i and j to 0 each, we get two equations in a and b, viz.

$$6 + \frac{9}{\sqrt{2}} - \frac{7}{\sqrt{2}} - 10 - \frac{5}{\sqrt{2}} + \frac{b}{\sqrt{2}} = 0 \qquad \text{...... (2)}$$

and $$\frac{9}{\sqrt{2}} + 8 + \frac{7}{\sqrt{2}} - \frac{5}{\sqrt{2}} - a - \frac{b}{\sqrt{2}} = 0 \qquad \text{...... (3)}$$

(2) gives $b=\sqrt{2}\left(\frac{3}{\sqrt{2}}+4\right)=3+4\sqrt{2}$ …… (4)

Putting this into (3) gives $a=8+\frac{1}{\sqrt{2}}(11-b)$

$$=8+\frac{1}{\sqrt{2}}\left(8-4\sqrt{2}\right)=4+4\sqrt{2} \quad \text{…… (5)}$$

From (4) and (5), $a+b=7+8\sqrt{2}$

Since $1.41<\sqrt{2}<1.42, 11.28<8\sqrt{2}<11.36$

So the integer closet to $8\sqrt{2}$ is 11 and hence that closest to $a+b$ is 7 + 11 = 18

17.(1) $b=0$ and $c=1$

Note that both x^3+2x^2+x+c and e^x are differentiable in their domain.

So make the function differentiable at $x = b$

Since f is differentiable at $x=b$

L.H.D. = R.H.D. (at $x = b$)

$3b^2+4b+1=e^b$

b is an integer, so LHS of the equation will always be an integer

However, RHS will be an integer only if $b = 0$

If $b=0$, LHS = RHS, so $b=0$ is the solution to this equation

Since f is differentiable at $x=b$, it is implied that it is also continuous at $x = b$

$\lim_{x\to b^-} f(x)=f(b)=\lim_{x\to b^+} f(x)$

$b^3+2b^2+b+c=e^0=1$

So the answer is 0 + 1 = 1

18.(3) Let us write $f(x)$ as $g(x)-h(x)$ where $g(x)=\frac{1}{2}x\sin x$

and $h(x)=1-\cos x=2\sin^2\left(\frac{x}{2}\right)$

$$\lim_{x\to 0}\frac{f(x)}{x^k}=\lim_{x\to 0}\frac{\frac{1}{2}x\sin x-1+\cos x}{x^k}=\lim_{x\to 0}\frac{\frac{1}{2}\sin x+\frac{1}{2}x\cos x-\sin x}{kx^{k-1}}$$

$$=\lim_{x\to 0}\frac{-\frac{1}{2}\cos x-\frac{1}{2}x\sin x+\frac{1}{2}\cos x}{k(k-1)x^{k-2}}=\lim_{x\to 0}\frac{-\frac{1}{2}x\sin x}{k(k-1)x^{k-2}}$$

Clearly $k=4$ and $\lambda=\frac{-1}{24} \quad \Rightarrow \quad k+24\lambda=3$